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## QCD corrections to charged Higgs-mediated $b \rightarrow c \tau \nu$ decay

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### Abstract

We calculate the  $O(\alpha_s)$  corrections to the charged Higgs mediated inclusive semi-tauonic  $B$  decay. By working in the Landau gauge, we demonstrate how to obtain the charged Higgs contributions (both direct and interference terms) from the known QCD corrections to  $t \rightarrow bH^+$ . Combining our results with  $O(1/m_b^2)$  corrections to the spectator model and comparing the theoretical prediction with the recent experimental measurements, we find a model-independent  $2\sigma$  upper bound on the ratio of Higgs vacuum expectation values,  $\tan \beta < 0.52 (m_H/1 \text{ GeV})$ .

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## I. INTRODUCTION

The  $B \rightarrow X \tau \nu$  decay is sensitive to extensions of the standard model (SM) Higgs sector. In particular, for two Higgs doublet models (2HDM) [1], the experimental measurement of this decay provides the strongest upper bound on  $\tan\beta/m_H$ , when the ratio of Higgs vacuum expectation values,  $\tan\beta$ , is large [2–4]. Since the charged Higgs boson contributes at tree level, its contribution cannot be canceled by other new particles in the theory. Consequently, this bound holds in all  $SU(2) \times U(1)$  electroweak models with a charged Higgs boson, independently of other details of the model. It is therefore important to calculate the charged Higgs contribution with high accuracy.

The leading order calculation is based on tree diagrams of the spectator model. Corrections to spectator model results are given by the  $1/m_b$  expansion in heavy quark effective theory (HQET) [5]. The  $O(1/m_b^2)$  corrections have been computed in the case of massless leptons [6–8], and for the tau-lepton in both the SM [9–11] and the 2HDM [4,12]. The one-loop QCD corrections to the tree-level amplitudes of the spectator model are equally important. In the SM, the  $O(\alpha_s)$  corrections have been computed for both massless leptons [13–15] and the tau-lepton [16–18]. However, no such calculations are available for the 2HDM. Their absence is a major source of uncertainty in the branching ratio predictions for large  $\tan\beta$  [4].

The  $O(\alpha_s)$  corrections for  $t \rightarrow bW$  [19] and  $t \rightarrow bH$  [20] are known. The QCD corrections for the  $B \rightarrow X \tau \nu$  decay can be directly obtained from these results. This method was applied in [17] to the SM calculation. In this paper, we show that by working in the Landau gauge, the method of [17] can be extended to include the charged Higgs contributions.

This paper is organized as follows. In section II we calculate the  $O(\alpha_s)$  corrections to the  $B \rightarrow X \tau \nu$  decay in the 2HDM (with model-II Higgs-fermion couplings [1]). In section III we combine this new result with the known  $O(1/m_b^2)$  corrections and the quark mass relations of HQET, and get an accurate prediction for the branching ratio as function of  $\tan\beta$  and the charged Higgs mass,  $m_H$ . Section IV contains our summary. Improvements to the QCD computation through renormalization group resummation are discussed in the Appendix.

## II. $O(\alpha_s)$ CORRECTIONS TO $B \rightarrow X \tau \nu$

In [17], the QCD corrections to  $B \rightarrow X \tau \nu$  in the SM were obtained using the known QCD corrections of  $t \rightarrow bW$  and  $t \rightarrow bH$  decays, and without calculating any new Feynman diagrams. The idea is to decompose the  $n$  body phase space (where  $n=3$  or 4) into a product of  $n-1$  body phase space and two-body phase space, integrated over the invariant squared mass of the  $\tau \nu$  pair [21]:

$$dPS(B \rightarrow X \tau \nu) \sim \int dq^2 dPS(b \rightarrow XW^*) dPS(W^* \rightarrow \tau \nu), \quad (2.1)$$

where  $q^\mu = p_\tau^\mu + p_\nu^\mu$  is the lepton pair momentum,  $X$  is either  $c$  or  $c$ +gluon, and  $dPS$  is the appropriate phase space differential element. The SM rate can be decomposed into two terms [17]: one corresponds to transverse  $W$  exchange and the other is equivalent to an

(un-physical) scalar exchange. The latter can be identified with the exchange of a massless Goldstone boson in the Landau gauge. When generalizing the analysis of [17] to include a charged Higgs boson, there are two additional terms: one that arises purely from the charged Higgs exchange and the other that arises from the interference between the  $W$  and the charged Higgs diagrams. The method proposed in [17] is now easily extended to this case by explicitly working in the Landau gauge. Then, there are three diagrams contributing to  $B \rightarrow X \tau \nu$  corresponding to  $W$ -exchange, Goldstone boson exchange, and charged Higgs boson exchange. It is easy to show that the rate can be decomposed into the sum of two incoherent decays:

$$\mathcal{M} = \mathcal{M}_W + \mathcal{M}_G + \mathcal{M}_H \quad \Longrightarrow \quad |\mathcal{M}|^2 = |\mathcal{M}_W|^2 + |\mathcal{M}_G + \mathcal{M}_H|^2. \quad (2.2)$$

The proof of (2.2) is straightforward. The lepton tensor in interference terms between a scalar (a Higgs or a Goldstone boson) and a vector boson ( $W$ ) carries one Lorentz index,  $L_\mu$ . After integration over the  $\tau \nu$  phase space,  $L_\mu$  must be proportional to the only Lorentz vector in the problem,  $q_\mu$  (the lepton pair momentum). Contracting  $L_\mu$  with the Landau gauge  $W$  propagator,  $T_{\alpha\mu} \sim g_{\alpha\mu} q^2 - q_\alpha q_\mu$ , we get

$$L^\mu T_{\alpha\mu} \propto q^\mu (g_{\alpha\mu} q^2 - q_\alpha q_\mu) = 0. \quad (2.3)$$

This proves (2.2). Note that this proof holds for arbitrary  $X$  in (2.1), and is therefore applicable to the QCD corrected rate to all orders in  $\alpha_s$  (at lowest order in the electroweak coupling expansion).

Let us define

$$\rho \equiv \frac{m_c^2}{m_b^2}, \quad \rho_\tau \equiv \frac{m_\tau^2}{m_b^2}, \quad t \equiv \frac{q^2}{m_b^2}, \quad \xi \equiv \frac{m_H^2}{m_b^2}. \quad (2.4)$$

Due to (2.2) we can write schematically (in the Landau gauge)

$$\Gamma(B \rightarrow X \tau \nu) = \int_{\rho_\tau}^{(1-\sqrt{\rho})^2} \frac{d\Gamma}{dt} dt, \quad (2.5)$$

$$\frac{d\Gamma}{dt} = H^{\alpha\beta} (b \rightarrow c W^*) T_{\alpha\mu} T_{\beta\nu} L^{\mu\nu} (W^* \rightarrow \tau \nu) + H (b \rightarrow c S^*) L (S^* \rightarrow \tau \nu),$$

where  $H$  and  $L$  refer to the terms that arise from analyzing the hadronic side and the leptonic side of the decay process, respectively. As above,  $T_{\alpha\mu}$  is the Landau gauge  $W$  propagator. The two terms on the right hand side of the above equation correspond to the sum over the two incoherent decays in (2.2). In particular,  $S$  stands for an effective scalar that we construct to give the contributions of both the Goldstone boson and the physical charged Higgs. Moreover, it is convenient to absorb the Goldstone boson and charged Higgs propagator factors ( $-1/q^2$  for the Goldstone boson and  $1/m_H^2$  for the physical Higgs boson in the approximation that  $q^2 \ll m_H^2$ ) directly into the definitions of the effective couplings of  $S$  to the leptons. As a result, the scalar propagator factors do not explicitly appear in (2.5). The couplings of  $S$  to quarks are then given by

$$\frac{ig m_b V_{cb}}{2\sqrt{2} m_W} (a + b \gamma_5), \quad (2.6)$$

where  $a \equiv a_G + a_H$ , and  $b \equiv b_G + b_H$ , with [1]

$$\begin{aligned} a_G &= -(1 - \sqrt{\rho}), & b_G &= -(1 + \sqrt{\rho}), \\ a_H &= \tan \beta + \sqrt{\rho} \cot \beta, & b_H &= \tan \beta - \sqrt{\rho} \cot \beta. \end{aligned} \quad (2.7)$$

The couplings of  $S$  to leptons (with the propagator factors included as indicated above) are given by

$$\frac{ig}{2\sqrt{2}m_W m_b}(a^\ell + b^\ell \gamma_5). \quad (2.8)$$

where  $a^\ell \equiv a_G^\ell + a_H^\ell$ , and  $b^\ell \equiv b_G^\ell + b_H^\ell$ , with [1]

$$\begin{aligned} a_G^\ell &= \frac{\sqrt{\rho_\tau}}{t}, & b_G^\ell &= \frac{-\sqrt{\rho_\tau}}{t}, \\ a_H^\ell &= \frac{\sqrt{\rho_\tau} \tan \beta}{\xi}, & b_H^\ell &= \frac{-\sqrt{\rho_\tau} \tan \beta}{\xi}. \end{aligned} \quad (2.9)$$

Integrating over the leptonic two-body phase space yields  $L$  [see (2.5)]

$$L(t) \sim \left(1 - \frac{\rho_\tau}{t}\right)^2 t \rho_\tau \left[ \frac{1}{t^2} - \frac{2 \tan \beta}{t \xi} + \frac{\tan^2 \beta}{\xi^2} \right]. \quad (2.10)$$

We denote  $\Gamma_{abc} \equiv \Gamma(a \rightarrow bc)$ . The expression for  $\Gamma_{tbH}$  can be found in [20]. For arbitrary quark flavors and scalar Yukawa couplings, Eqs. (7) and (8) in [20] can be parameterized as  $\Gamma_{QqS}(m_q^2/m_Q^2, m_S^2/m_Q^2; c_1, c_2, c_3)$ . The relevant combinations of couplings are

$$\begin{aligned} c_1 &= a^2 + b^2 = 2(1 + \rho) + 2(\tan^2 \beta + \rho \cot^2 \beta) - 4(\tan \beta - \rho \cot \beta), \\ c_2 &= a^2 - b^2 = 4\sqrt{\rho}[-1 + \tan \beta \cot \beta + (\tan \beta - \cot \beta)], \\ c_3 &= ab = (1 - \rho) + (\tan^2 \beta - \rho \cot^2 \beta) - 2(\tan \beta + \rho \cot \beta). \end{aligned} \quad (2.11)$$

Then, the contribution to the  $B \rightarrow X \tau \nu$  decay rate from the effective scalar  $S$  is obtained by taking  $S$  to be off-shell and integrating over its momentum:

$$\Gamma_S(B \rightarrow X \tau \nu) \sim \int_{\rho_\tau}^{(1-\sqrt{\rho})^2} \Gamma_{bcS}(\rho, t; c_1, c_2, c_3) L(t) dt, \quad (2.12)$$

where  $L(t)$  is given in (2.10) and  $c_1, c_2$  and  $c_3$  are given in (2.11).

### III. NUMERICAL ANALYSIS

In our numerical analysis we set  $\cot \beta = 0$ , as we are always interested in large  $\tan \beta$ , and we neglect the tiny  $b \rightarrow u$  transitions. Since  $\Gamma_{QqS}$  is linear in the three relevant terms in (2.11) we can identify the Higgs and the interference terms in the total rate

$$\begin{aligned}\Gamma_H(B \rightarrow X \tau \nu) &= \frac{G_F m_b^2}{4\sqrt{2}\pi} \int_{\rho_\tau}^{(1-\sqrt{\rho})^2} \Gamma_{bcS}(\rho, t; 2, 0, 1) \left(1 - \frac{\rho_\tau}{t}\right)^2 \frac{t \rho_\tau \tan^4 \beta}{\xi^2} dt, \\ \Gamma_I(B \rightarrow X \tau \nu) &= -\frac{G_F m_b^2}{2\sqrt{2}\pi} \int_{\rho_\tau}^{(1-\sqrt{\rho})^2} \Gamma_{bcS}(\rho, t; 2, -2\sqrt{\rho}, 1) \left(1 - \frac{\rho_\tau}{t}\right)^2 \frac{\rho_\tau \tan^2 \beta}{\xi} dt,\end{aligned}\quad (3.1)$$

where  $\Gamma_H$  ( $\Gamma_I$ ) is the Higgs (interference) term in  $\Gamma$ . The tree level decay rate can be obtained by inserting the appropriate tree-level expression for  $\Gamma_{bcS}$  into (3.1). We have performed the integrals analytically and checked that we recover the known results [3,4]. Turning to the  $O(\alpha_s)$  computation, we insert the appropriate one-loop expressions for  $\Gamma_{bcS}$  deduced from [20]. The integrals in (3.1) are then evaluated numerically.

Our results are not very sensitive to small variations in the choice of the  $c$  and  $b$  pole masses. Below we quote results for representative values of the pole masses  $m_c = 1.4$  GeV and  $m_b = 4.8$  GeV. Decomposing the total rate into the tree-level and the one-loop parts,

$$\Gamma = \Gamma^0 + \alpha_s \Gamma^1, \quad (3.2)$$

it is convenient to quote results for  $\Gamma^1/\Gamma^0$  (note that factors of  $\tan \beta$  cancel in the ratio). We get

$$\frac{\Gamma_H^1}{\Gamma_H^0} = -0.753, \quad \frac{\Gamma_I^1}{\Gamma_I^0} = -0.487. \quad (3.3)$$

The SM corrections were calculated in [11] based on the calculations of [16], and in [17]:

$$\frac{\Gamma_W^1}{\Gamma_W^0} = -0.450, \quad \frac{\Gamma_e^1}{\Gamma_e^0} = -0.545, \quad (3.4)$$

where  $\Gamma_W$  ( $\Gamma_e$ ) is the  $W$  mediated rate for  $B \rightarrow X \tau \nu$  ( $B \rightarrow X e \nu$ ). Defining  $\eta \equiv \Gamma/\Gamma^0$ , and using  $\alpha_s(m_b) \simeq 0.22$  [corresponding to  $\alpha_s(m_Z) = 0.115$ ], we get

$$\eta_e \simeq 0.88, \quad \eta_W \simeq 0.90, \quad \eta_I \simeq 0.89, \quad \eta_H \simeq 0.83. \quad (3.5)$$

The main uncertainties in the calculation are from the unknown  $O(\alpha_s^2)$  corrections. To account for these, we vary  $\alpha_s$  in the range  $0.20 \leq \alpha_s(\mu) \leq 0.36$ , corresponding to  $m_b/3 \leq \mu \leq m_b$  and  $0.110 \leq \alpha_s(m_Z) \leq 0.125$ . This leads to the ranges

$$1.02 \leq \frac{\eta_W}{\eta_e} \leq 1.04, \quad 1.01 \leq \frac{\eta_I}{\eta_e} \leq 1.03, \quad 0.96 \geq \frac{\eta_H}{\eta_e} \geq 0.91. \quad (3.6)$$

Note that these ratios are rather insensitive to the exact choice of  $\alpha_s$ . This completes the analysis of the  $O(\alpha_s)$  corrections.

To arrive at our final prediction, one must also include the  $O(1/m_b^2)$  corrections to the spectator model. In the Heavy Quark Expansion, we use the HQET mass relation

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \dots, \quad m_D = m_c + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_c} + \dots, \quad (3.7)$$

and the numerical values as in [4]

$$0.4 < \bar{\Lambda} < 0.6 \text{ GeV}, \quad 0 < -\lambda_1 < 0.3 \text{ GeV}^2, \quad 0.11 < \lambda_2(m_b) < 0.13 \text{ GeV}^2. \quad (3.8)$$

Combining the  $O(1/m_b^2)$  calculation of [4] with our  $O(\alpha_s)$  calculation, and using the range of the various parameters as given in Eqs. (3.6) and (3.8), we get the theoretical prediction for the branching ratio (normalized to the electron channel) as a function of  $r \equiv \tan \beta / m_H$  given in Fig. 1. Comparing it with Fig. 1(a) of [4] we learn that, as anticipated in [4], the errors in the plot are reduced by about a factor of two for large  $r$ . The residual errors arise almost entirely from the unknown pole masses, namely the values of  $\bar{\Lambda}$  and  $\lambda_1$ .

Combining the recent measurements of the branching ratio of  $B \rightarrow X \tau \nu$  from ALEPH [22] and L3 [23] and using the world average for  $B \rightarrow X \ell \nu$  ( $\ell = e, \mu$ ) [24],

$$\text{BR}(B \rightarrow X \tau \nu) = 2.69 \pm 0.44 \%, \quad \text{BR}(B \rightarrow X \ell \nu) = 10.43 \pm 0.24 \%, \quad (3.9)$$

we obtain the  $2\sigma$  upper bound

$$r < 0.52 \text{ GeV}^{-1}. \quad (3.10)$$

The  $1\sigma$  upper bound is  $r < 0.49 \text{ GeV}^{-1}$  (as compared to  $r < 0.51 \text{ GeV}^{-1}$  in [4]). The improvement over the bounds obtained in [4] originates from both the experimental improvement and from our  $O(\alpha_s)$  calculation.

Our analysis holds with minor modifications for general multi-Higgs doublet models with natural flavor conservation (for a recent analysis, see [25]). In such models, instead of the single parameter  $\tan \beta$ , three complex coupling constants determine the Yukawa interactions of the lightest charged scalar (assuming that the heavier charged scalars effectively decouple from the fermions). The parameters  $X$ ,  $Y$  and  $Z$  describe the couplings to down-type quarks, up-type quarks and charged leptons, respectively. The bounds on  $\sqrt{|XZ^*|}/m_H$  and  $\sqrt{|\text{Re}(XZ^*)|}/m_H$  are the same as that on  $r$ . The  $2\sigma$  upper bound on the imaginary part of  $XZ^*$  is  $\sqrt{|\text{Im}(XZ^*)|}/m_H < 0.44 \text{ GeV}^{-1}$  ( $< 0.41 \text{ GeV}^{-1}$  at  $1\sigma$ ).

#### IV. SUMMARY

We have calculated the  $O(\alpha_s)$  corrections to  $B \rightarrow X \tau \nu$  decay in the 2HDM. In addition to the known SM result, we have computed the contributions arising from the exchange of a charged Higgs boson; both the pure Higgs term and the  $W^+ - H^+$  interference term have been included. Working in the Landau gauge, the calculation is carried out by making use of previous calculations of the QCD corrections to  $t \rightarrow bH$ . Our results for the numerical value of the QCD corrections are given in Eq.(3.5). We combined our calculation with the known  $O(1/m_b^2)$  corrections and the quarks mass relations of HQET, and obtained an accurate prediction for the branching ratio as function of  $\tan \beta / m_H$ , shown in Fig. 1. The remaining uncertainties in the prediction are almost entirely from the uncertainties in the pole masses.

The virtual effects of the charged Higgs boson can be important in other processes. However, in most cases, the charged Higgs exchange only first enters at the one-loop level. In models with an extended Higgs sector, there is often additional new physics beyond the Standard Model that can also contribute to the processes under consideration. In this case, there can be no clean bound on Higgs sector parameters, since the one-loop charged Higgs

effects can be partially canceled by the effects of other particles in the theory. For example, in the minimal supersymmetric model, the bounds on the charged Higgs parameters from the one-loop charged Higgs contributions to  $b \rightarrow s\gamma$  [26] and  $Z \rightarrow b\bar{b}$  [27] decays can be avoided if partially canceled by loop diagrams with intermediate superparticles. In contrast, the charged Higgs boson contribution to  $B \rightarrow X \tau \nu$  studied in this paper occurs at tree level; therefore, its effect cannot be canceled by other new particles in the theory.

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## APPENDIX A: RENORMALIZATION GROUP IMPROVEMENT

The QCD corrections to  $\Gamma(t \rightarrow bH)$  were calculated for  $m_b = 0$  in [28,29], and extended for finite  $m_b$  in [20,30]. Hence, we use the results of [20]\*. All calculations were performed in the on-shell mass renormalization scheme, in which the quark pole masses appear in the formulae. These results can be improved by using the technology of the renormalization group equations (RGE) to resum the leading logarithms to all orders in  $\alpha_s$ . In particular, note that for small  $\epsilon \equiv m_b^2/m_t^2$ ,  $\alpha_s \log \epsilon$  is typically of  $O(1)$ . Consequently, all leading logarithmic terms of the form  $\epsilon \alpha_s^n \log^n \epsilon$  should be summed [31]. This can be accomplished by noting that one of the effects of the one-loop QCD corrections is the replacement of the pole mass  $m_b$  with the one-loop running mass  $m_b(m_t)$  [32]. By replacing the one-loop  $m_b(m_t)$  with the fully RGE-integrated running mass,  $m_b^{RGE}(m_t)$ , one successfully resums all  $\alpha_s \log \epsilon$  terms.

The one-loop QCD corrections to  $\Gamma(t \rightarrow bH)$  were obtained in [20], but the RGE-improved rate was not given. To carry out the RGE-improvement, first expand the results of [20] in powers of  $\epsilon$ . Formally, the full QCD-corrected rate can be written as

$$\Gamma = \sum_{nm} \epsilon^n \alpha_s^m \Gamma_n^m, \quad (\text{A1})$$

which defines  $\Gamma_n^m$ . In this case, only terms up to  $O(\alpha_s)$  are known. It is sufficient to keep terms up to  $O(\epsilon)$  since  $\epsilon \simeq 10^{-3}$  is a small number, and only the  $O(\epsilon)$  term can be enhanced by the possibly large  $\tan^2 \beta$  coefficient. Thus, we write  $\Gamma = \Gamma_0^0 + \alpha_s \Gamma_0^1 + \epsilon (\Gamma_1^0 + \alpha_s \Gamma_1^1)$ . Using the expressions given in [20], we find

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\*The results of [30] differ from those of [20]. However, the latter results appear more reliable; see the discussion of [20] for further clarifications.

$$\Gamma = \Gamma_0^0 + \epsilon \Gamma_1^0 \left( 1 + \frac{4\alpha_s}{\pi} \log \epsilon \right) + \alpha_s \Gamma_{fin}^1, \quad (\text{A2})$$

where  $\Gamma_{fin}^1$  contains no  $\log \epsilon$  terms, and

$$\begin{aligned} \Gamma_0^0 &= \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi} (1 - \chi) \cot^2 \beta, \\ \Gamma_1^0 &= \frac{G_F m_t^3 |V_{tb}|^2}{2\sqrt{2}\pi} \left\{ (1 - \chi) \left[ 1 + \frac{1}{4}(1 - \chi) \tan^2 \beta \right] - \frac{\chi}{2} \cot^2 \beta \right\}, \end{aligned} \quad (\text{A3})$$

where  $\chi \equiv m_H^2/m_t^2$ . Using

$$m_b^2(m_t) = m_b^2(m_b) \left( 1 + \frac{4\alpha_s}{\pi} \log \epsilon + O(\alpha_s^2) \right), \quad (\text{A4})$$

we see that (A2) is equivalent at  $O(\alpha_s)$  to replacing  $m_b(m_b)$  with  $m_b(m_t)$  in  $\Gamma_0^0 + \epsilon \Gamma_1^0$ . This is a general result [32]. Then, to sum up the  $\epsilon \alpha_s^n \log^n \epsilon$  terms, we replace the  $O(\alpha_s)$  expression for  $m_b(m_t)$  with  $m_b^{RGE}(m_t)$  [31]:

$$m_b(m_t) = m_b(m_b) \left( 1 + \frac{2\alpha_s}{\pi} \log \epsilon \right) \implies m_b^{RGE}(m_t) = m_b(m_b) \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right]^{-2\gamma_0/\beta_0}, \quad (\text{A5})$$

where  $\gamma_0 = 2$  and  $\beta_0 = 11 - 2/3n_f$ .

The decay rate depends on  $m_b$  through both the Yukawa coupling and the phase space integrals. Far from threshold, the dependence of the phase space integrals on  $m_b$  is weak; therefore, the RGE-improvement has no numerical significance there. In contrast, the RGE-improvement via the Yukawa couplings is important provided that  $\tan \beta$  is large (it is only for large  $\tan \beta$  that the rate is sensitive to  $m_b$ ). To see the effect, we define  $\eta \equiv \Gamma/\Gamma^0$  prior to RGE-improvement (see section III), where in the notation above,  $\Gamma^0 \equiv \sum_n \epsilon^n \Gamma_n^0$ . The results of the RGE-improvement are sensitive to the procedure that we take. We can use the RGE-improved  $m_b(m_t)$  in  $\Gamma^0$  only (we call the result  $\eta_0^{RGE}$ ) or in both  $\Gamma^0$  and  $\Gamma_{fin}^1$  ( $\eta_1^{RGE}$ ). The difference is  $O(\alpha_s^2)$ , beyond the accuracy of the calculation, but in practice it can turn out to be important. We take  $m_b = 4.8$  GeV,  $m_t = 175$  GeV,  $m_H = 100$  GeV and  $\alpha_s(m_t) = 0.1$ . As a first example, take  $\tan \beta = 1$ . Then  $\eta \sim \eta_0^{RGE} \sim \eta_1^{RGE} \sim 0.92$ , with difference of  $O(10^{-4})$ . We see that the weak dependence of the rate on  $m_b$  makes the RGE-improvement unimportant. As a second example, take  $\tan \beta = 100$ :

$$\eta \sim 0.46, \quad \eta_0^{RGE} \sim 0.50, \quad \eta_1^{RGE} \sim 0.46. \quad (\text{A6})$$

The difference between  $\eta_0^{RGE}$  and  $\eta_1^{RGE}$  is a measure of the size of the neglected  $O(\alpha_s^2)$  terms. Given that  $\eta \simeq \eta_1^{RGE}$ , it follows that for very large  $\tan \beta$ , the RGE-resummed result does not provide a practical improvement over the original one-loop calculation.

The situation is different for the  $B \rightarrow X \tau \nu$  decay in three important ways:

(i) The RGE improvement is related to the Yukawa coupling of the light quark. In  $B \rightarrow X \tau \nu$ , the Yukawa coupling of the charm can be safely neglected.

(ii) The RGE improvement is important to sum large logs. However,  $\log(m_b/m_c) \sim O(1)$  is not large.



(iii) The RGE improvement is a correction to the  $O(\alpha_s)$  corrections, but as noted below (3.6), the latter almost cancel in the relevant ratio of rates that we consider.

Therefore, while theoretically important, the RGE improved result has no numerical significance for the  $B \rightarrow X \tau \nu$  calculation.

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# FIGURES

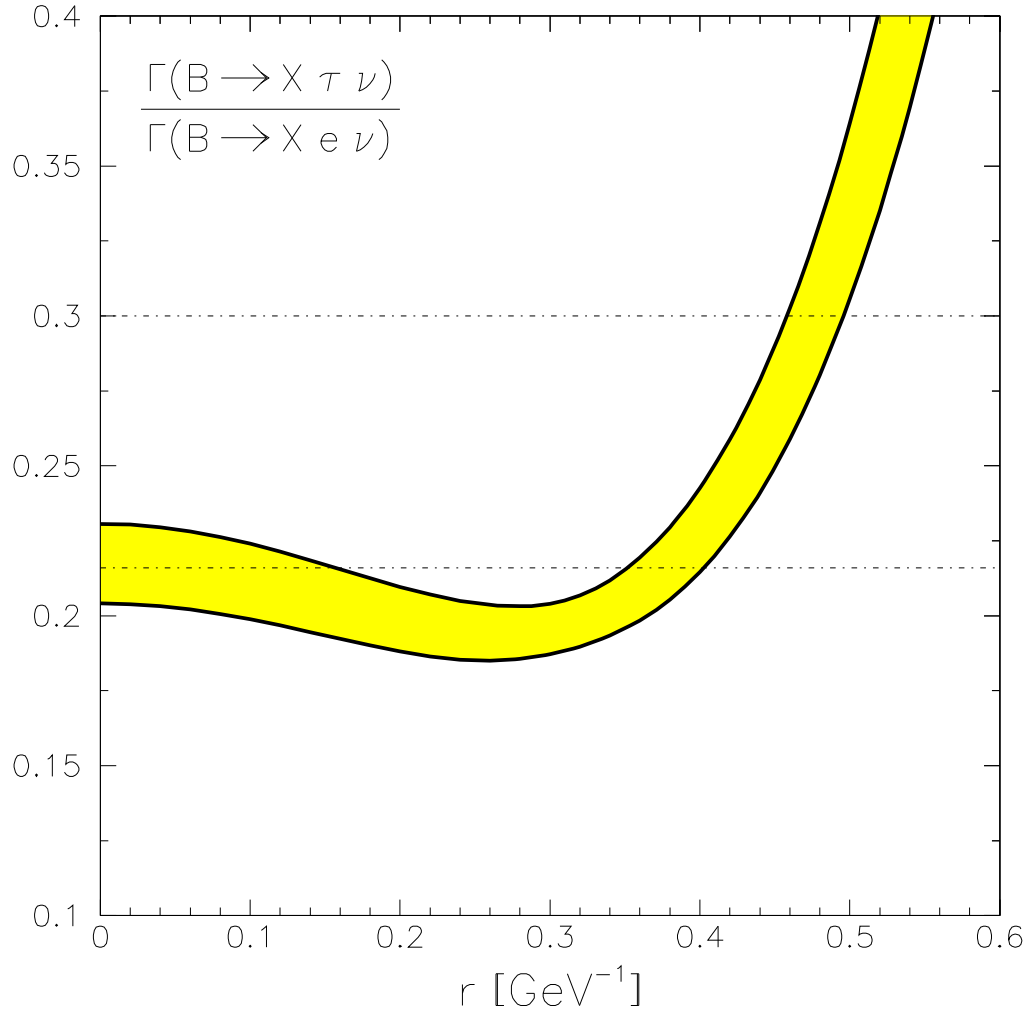


FIG. 1.  $\Gamma(B \rightarrow X \tau \nu)/\Gamma(B \rightarrow X e \nu)$  as a function of  $r = \tan \beta/m_H$ . The shaded area between the solid lines is our result. The dash-dotted lines give the experimental  $1\sigma$  bounds.